Advanced Image Denoising Methods: TV, NLM, and BM3D

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Outline

- TV [Rudin, et al. 1992]:

  Total Variation minimization

- NLM [Buades, et al. 2005]:

  Non-Local Means

- BM3D [Dabov, et al. 2006/2007]:

  Block-Matching and 3D filter

Total Variation minimization (TV)

$$g(x,y) = f(x,y) + n(x,y)$$

Observed image

Clean Image

Additive noise

$$\hat{f} = g \otimes h$$

Smooth kernel (local method)

Total Variation minimization (TV)

Global method:

$$TV\left(\hat{f}\right)=\sum_{x,y}\sqrt{\left(\hat{f}(x+1,y)-\hat{f}(x,y)\right)^2+\left(\hat{f}(x,y+1)-\hat{f}(x,y)\right)^2}$$

$$=\begin{pmatrix} \boxed{124} & 100 & 30 \\ 69 & 80 & 200\cdots \\ 66 & 92 & 211 \\ & \vdots & \end{pmatrix}$$

Lower TV →

Total Variation minimization (TV)

$$\arg\min_{\hat{f}} \underbrace{\frac{1}{2}\left\| g - \hat{f} \right\|_2^2}_{\text{Fidelity term}} + \lambda \cdot TV\left(\hat{f}\right)$$

f =

g

$\hat{f}$, λ = low

$\hat{f}$, λ = high

$\hat{f}$, $\lambda = \infty$

Total Variation minimization (TV)

Figure 1. An image.

σ = 12

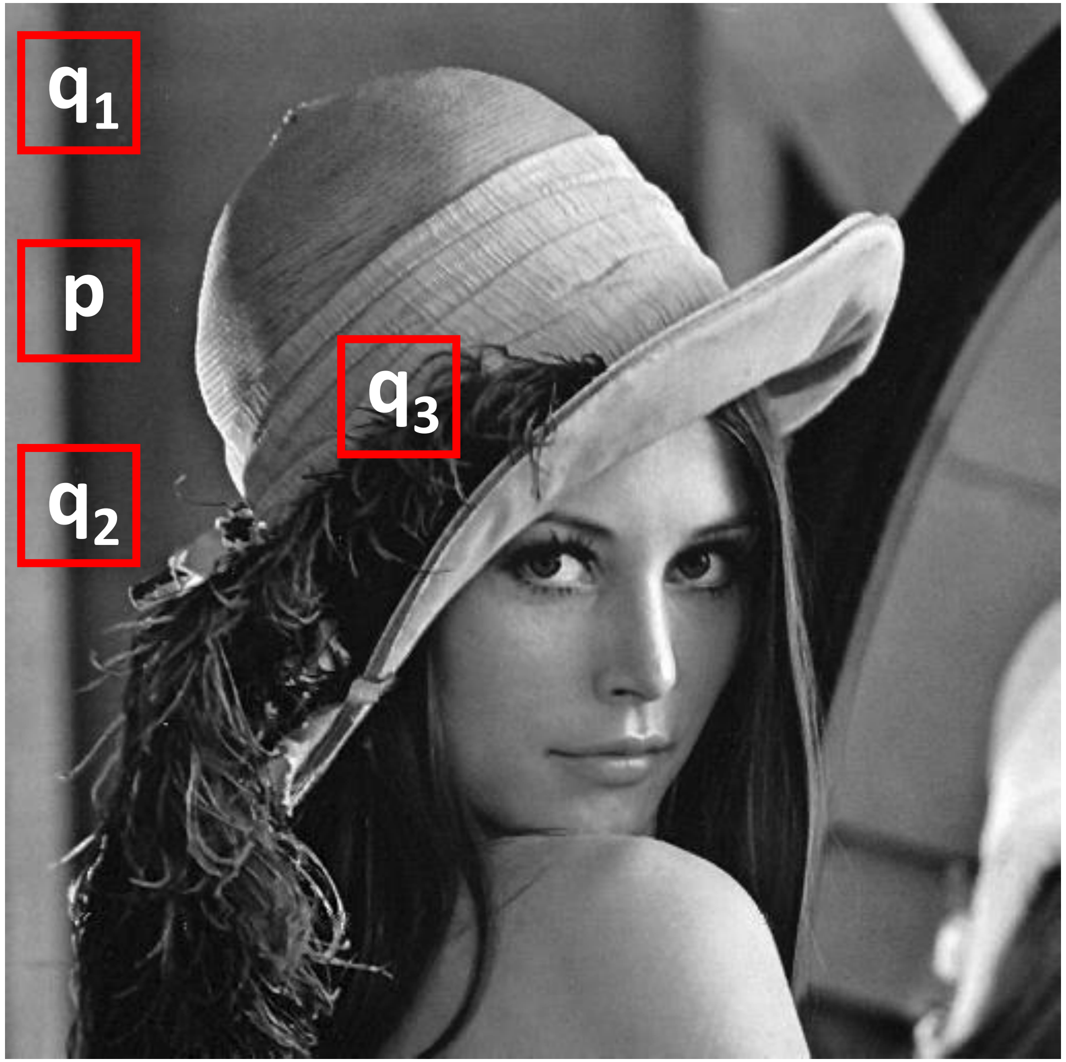
σ = 25

Figure 2. The image of Fig. 1 and its reconstruction ($\sigma = 12$).

Figure 3. Same as Fig. 2 with now $\sigma = 25$.

- **Straight edges are maintained.**
- **Details and texture can be over smoothed if λ is too large.**

Non-Local Means (NLM)



$$NLM(x,y)=\sum_{i,j} w_{xy}(i,j)\,g(i,j)$$

$$0 \le w_{xy}(i,j) \le 1,\ \sum_{i,j} w_{xy}(i,j)=1$$

$$w_{xy}(i,j)=\frac{\exp\left(-\frac{\left\| g(\mathrm{N}_{xy})-g(\mathrm{N}_{ij})\right\|_2^2}{\sigma^2}\right)}{\sum_{ij}\exp\left(-\frac{\left\| g(\mathrm{N}_{xy})-g(\mathrm{N}_{ij})\right\|_2^2}{\sigma^2}\right)}$$

Non-Local Means (NLM)

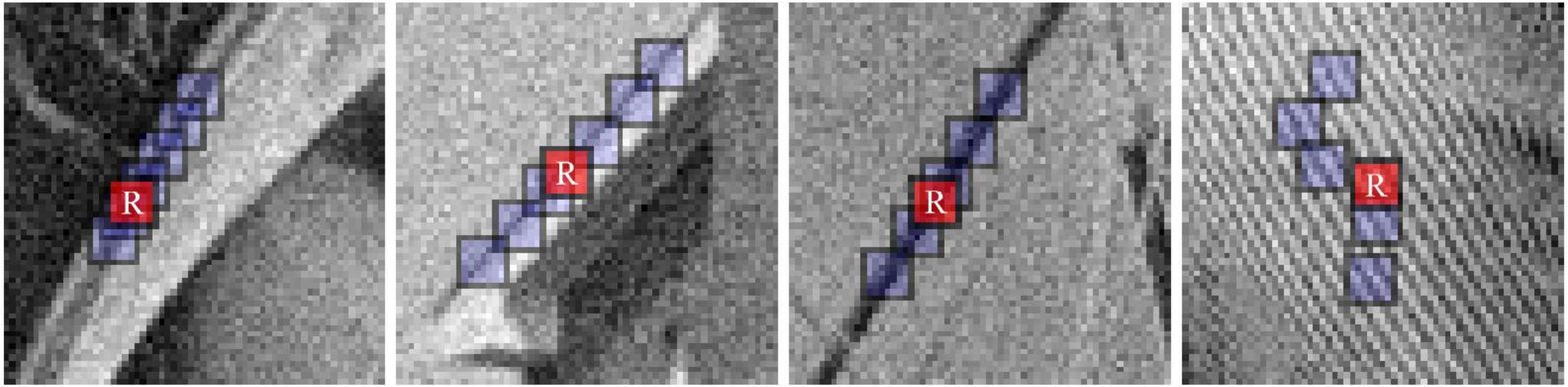
Local | Global | Non-Local

Noisy image $\sigma = 20$ | Gaussian kernel | TV | NLM

- **Preserve straight edges, as well as details and texture.**

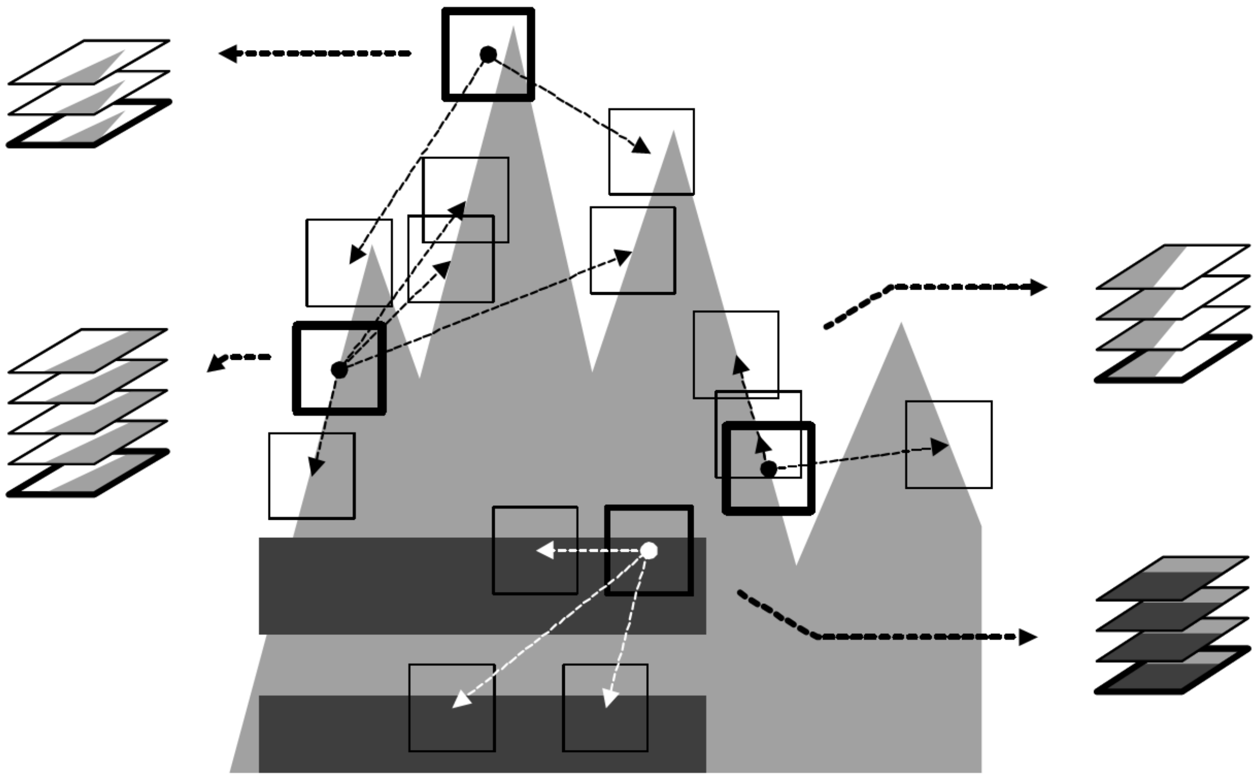
Block-Matching and 3D filter (BM3D)

Block matching + 3D transform



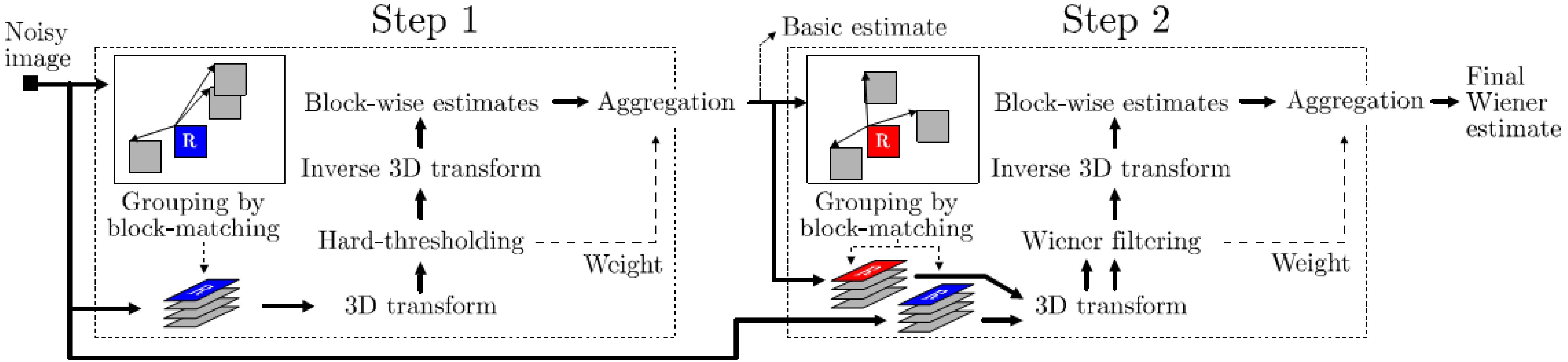
Block-Matching and 3D filter (BM3D)

Block matching + 3D transform



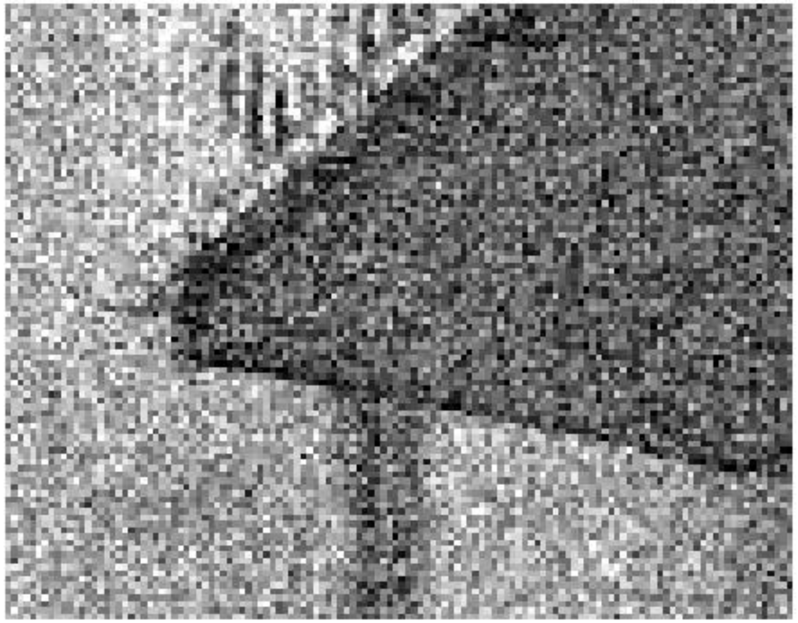
- Element-wise averaging
  - Identical blocks
  - Multiple blocks
- 3D transform (e.g., DWT, DFT, DCT)

Block-Matching and 3D filter (BM3D)

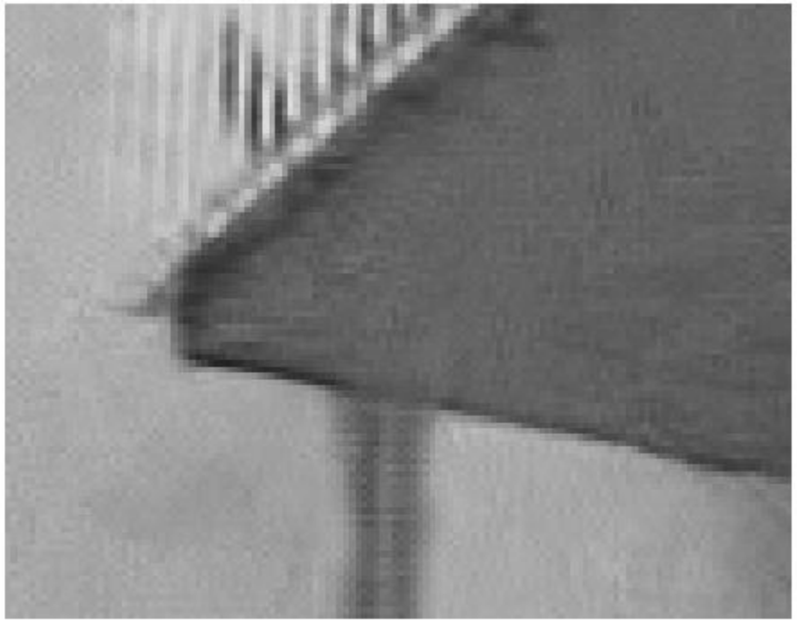


- Using the basic estimate instead of the noisy image allows to improve the grouping by block-matching.
- Using the basic estimate as the pilot signal for the empirical Wiener filtering is much more effective and accurate than the simple hard-thresholding.

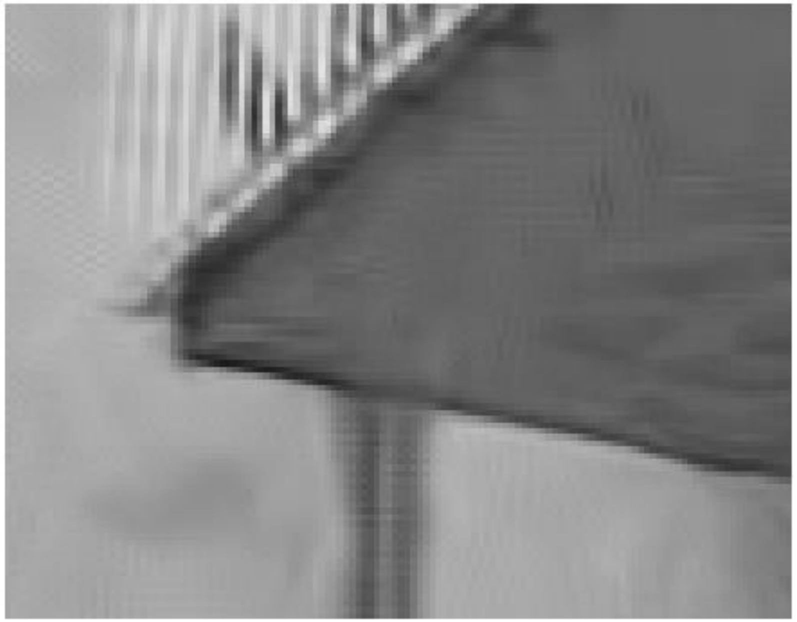
Block-Matching and 3D filter (BM3D)



Noisy image $\sigma = 40$



Basic estimate



Final estimate

Block-Matching and 3D filter (BM3D)

Thank You